Serie 06 - Solution

Preamble

In Lecture 6, we introduced the Bipolar Junction Transistor (BJT) and its equations. Many approximations and assumptions are made during the demonstrations, and we recall them here. In order to understand them and the BJT in general, it is important to get familiar with the notation that indicates different positions along the device length. We report below the minority carrier profiles of a BJT with short base, emitter, and collector in forward operation, as an example. The focus is on understanding the meaning of the different positions along the x-axis.
Back to the assumptions and approximations:

1. **Short base approximation.** A well-designed BJT has a very short base (compared to the electron diffusion length): $W_B^{'} = W_B - X_{pE} - X_{pC} << L_{nB}$. In forward operation, this ensures minimal recombination of electrons injected into the base from the emitter, which are then accelerated by the electric field in the depleted region of the base-collector junction (reversely biased) and easily collected by the collector. This ensures a high gain of the BJT. This is always the case for a good BJT. This entails that the minority carrier profile in the QNR of the base is always approximately linear with x (see Preamble and Solutions of Serie 05) and depends on $W_B^{'}$, and not on L_{nB}.

2. **Influence of the base depleted regions.** As we saw in Serie 05, in the short neutral side case, the minority carrier profiles depend on the side length W minus the depleted region on that side X_p (or X_n). We also know that the extension of the depleted region depends on the bias applied to the junction. This is what causes in a BJT the Early effect, for example, where the collector current I_C changes as a function of V_{CE} because this modulates the width X_{dC}, therefore $W_B^{'}$, and therefore the common base current amplification in forward operation β_F. In some cases, the width of the base depleted regions will be neglected so that $W_B^{'} \sim W_B$. This approximation has to be verified by calculating the widths of the depleted regions at the specific operation point.

3. **Emitter and Collector neutral sides lengths.** Once again, depending on the length of the emitter and collector neutral sides compared to the diffusion length of holes (for an NPN transistor), the holes minority carrier profile can be exponential or linear in the QNRs. The formulas for the diffusion currents have to be adjusted accordingly. In particular, the common base current amplification in forward operation α_F has to be adjusted to consider $W_E^{'}$ or L_{pE}.

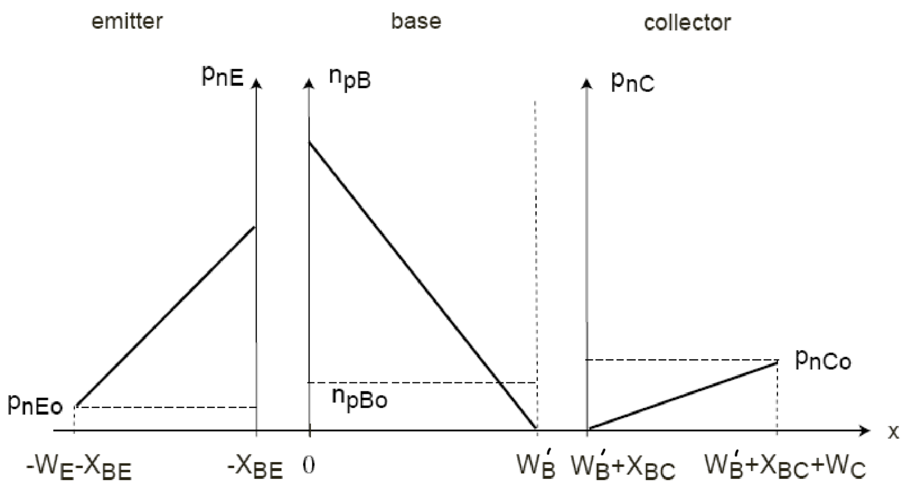


Figure 1: Minority carriers profiles in a NPN BJT in forward bias.

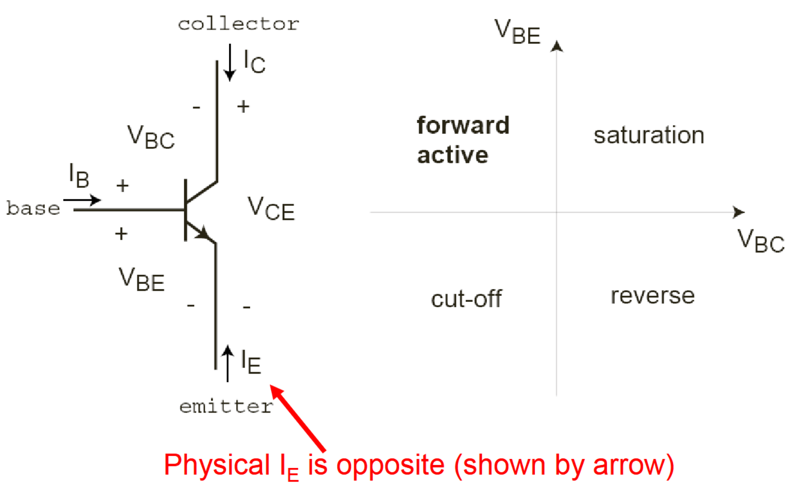


Figure 2: NPN BJT: circuit symbols and operating regions.

Exercise 01

Consider the properties associated to the polarization of a bipolar junction transistor (BJT) in different regions: **forward active, reverse, saturation, cut-off**.
Among the following sentences, select the correct ones.

Q1. The BJT can be used in analogical amplification applications both in forward active region and in reverse region.

Sol: **False.** The BJT transistors are usually optimized to have high gain only in the active region. Reverse bias gain is usually very small to be of use for circuit applications.

Q2. The amplification factor β is independent on the polarization region.

Sol: **False.** See explanation of A. BJT can only work as an amplifier in the forward active region.

Q3. From a circuital point of view, a BJT is equivalent to two junction diodes.

Sol: **False.** The two junctions are put so close that the minority carriers can actually travel from the emitter to the collector with very little recombination, one cannot think of BJT as two back-to-back diodes due to this interaction.

Q4. The saturation region is interesting for an application as an electronic switch, due to the low resistance.

Sol: **True.**

Q5. The large difference in the values of β in the forward and reverse regions is due to the difference in doping between the emitter and the collector (the transistor is optimized only for the forward active region).

Sol: **True.** See Q1.

Q6. In saturation, we can approximate $V_{CE} \sim 0$ V.

Sol: **True.** See Q4.

Q7. The values of the factor $\beta = I_C/I_B$ in the saturation and forward active regions are identical.

Sol: **False.** In the saturation region there is no current gain.

Q8. The values of the factor β in the forward active and reverse regions for the same transistor can be equal, if the BJT has perfectly symmetric doping of the emitter and collector.

Sol: **True.** If every parameter is kept the same between the emitter and the collector, one can switch between the two terminals and obtain the same device operation.

Q9. The slope of the characteristics I_C and I_B as a function of V_{BE} is limited to 60 mV/dec at T=300 K, as in the junction diode.

Sol: **True.** Since BJT also relies on the diffusion currents for operation, it is similarly bound by the 60 mV/dec slope.

Exercise 02

Calculate the relationship between the current amplification factor $\beta = I_C/I_B$ and the factor $\alpha = I_C/I_E$ of a BJT in forward active region. Find the values of β for $\alpha = 0.95$ and 0.99.

Solution

$$I_E = I_C + I_B$$

$$\frac{I_C}{\alpha} = I_C + \frac{I_C}{\beta} \Leftrightarrow \frac{1}{\alpha} = 1 + \frac{1}{\beta}$$

$$\alpha = \frac{\beta}{1+\beta} \Leftrightarrow \beta = \frac{\alpha}{1-\alpha}$$

For $\alpha = 0.95$: $\beta = \frac{0.95}{1-0.95} = 19$.
For $\alpha = 0.99$: $\beta = \frac{0.99}{1-0.99} = 99$.

Exercise 03

Determine the ratio between the doping concentration of the emitter N_{DE} and that of the base N_{AB} in an NPN BJT, so that the common base current amplification in forward operation is $\alpha_F = 0.9967$. What is the physical meaning of α_F? How can the common emitter current amplification in forward operation β_F be improved technologically?
For this exercise, we suppose $D_p \approx D_n$ and $W_B \approx W_E$, with W_B and W_E much smaller than the minority electrons and holes diffusion lengths. We also neglect the width of the depletion region of the B-E junction.

Solution

In the previous exercise, we found the relation between the two factors α_F and β_F, that we can write in the following way:

$$\alpha_F = \frac{\beta_F}{1+\beta_F} = \frac{1}{1+\frac{1}{\beta_F}} \tag{1}$$

We now write the explicit formula for β_F considering the assumptions made in the assignment: 1) negligible depletion regions, which entails $W'_B \approx W_B$; 2) short neutral side assumption for the base and the emitter, for which we need to consider W as the critical dimension for the diffusion of minority carriers. That said, we can write:

$$\beta_F = \frac{I_C}{I_B} = \frac{n_{pB0}\frac{D_n}{W_B}}{p_{nE0}\frac{D_p}{W_E}} = \frac{N_{DE}D_nW_E}{N_{AB}D_pW_B} \tag{2}$$

with $n_{pB0} = \frac{n_i^2}{N_{AB}}$ and $p_{nE0} = \frac{n_i^2}{N_{DE}}$.
Since we also have $D_p \approx D_n$ and $W_B \approx W_E$, we can simplify and obtain:

$$\alpha_F = \frac{1}{1+\frac{1}{\beta_F}} = \frac{1}{1+\frac{N_{AB}}{N_{DE}}} = 0.9967 \Rightarrow \frac{N_{AB}}{N_{DE}} \approx 3.3\cdot 10^{-3} \tag{3}$$

Comment: The emitter injection efficiency α_F represents the ratio between the "useful" current/carriers injected by the emitter that contribute to the collector current and the total emitter current, including the carriers injected by the base into the emitter that do not contribute to the collector current. To improve α_F, and consequently β_F, the emitter must be highly doped, much more than the base.

Exercise 04

Calculate the change in the neutral base width W'_B of a bipolar transistor with the voltage V_{BC}. Assume uniform doping concentrations $N_{AB} = 5 \cdot 10^{16} \left[cm^{-3}\right]$ and $N_{DC} = 2 \cdot 10^{15} \left[cm^{-3}\right]$, the transistor at room temperature $T = 300\,[K]$ and in forward operation region. The metallurgical dimension of the base is $W_B = 0.70\,[um]$. The voltage V_{BC} changes from $-2\,[V]$ to $-10\,[V]$. By how much, in %, does the neutral base width change when V_{BC} has the proposed variation? Discuss the impact of this observation on the collector current and the definition of the Early voltage of the bipolar transistor.

Solution

We start by calculating the width of the depletion region in the base at the B-C junction. We are defining it $x_{p,BC}$ for these calculations. We just need to calculate the built-in potential first:

$$\phi_b = \frac{kT}{q} \ln\left(\frac{N_{AB}N_{DC}}{n_i^2}\right) = 697\,[mV] \tag{4}$$

$$x_{p,BC}\left(V_{BC}\right) = \sqrt{\frac{2\epsilon_{Si}N_{DC}}{qN_{AB}(N_{AB}+N_{DC})}\left(\phi_b - V_{BC}\right)} \approx 3.155 \cdot 10^{-6}\sqrt{\phi_b - V_{BC}}\left[\frac{cm}{\sqrt{V}}\right] \tag{5}$$

$$V_{BC} = -2\,[V] \Rightarrow x_{p,BC} \approx 0.052\,[um]$$

$$V_{BC} = -10\,[V] \Rightarrow x_{p,BC} \approx 0.103\,[um]$$

Since the BJT is in forward operation, the B-E junction is forward biased, contrary to the B-C junction. Therefore, we assume the depletion region at the B-E junction is really thin and that we can neglect it when calculating W'_B. This said, the neutral base widths are respectively:

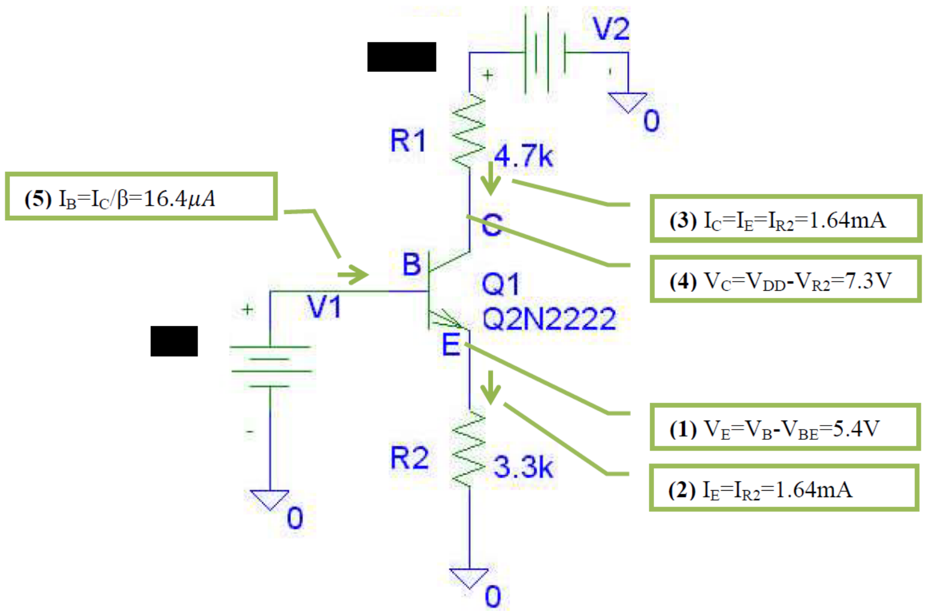
$$V_{BC} = -2\,[V] \Rightarrow W'_B = (0.700 - 0.052)\,[um] = 0.648\,[um]$$

$$V_{BC} = -10\,[V] \Rightarrow W'_B = (0.700 - 0.103)\,[um] = 0.597\,[um]$$

which is equivalent to a change of $\approx 8\%$ as V_{BC} changes from $-2\,[V]$ to $-10\,[V]$.

Exercise 05

Consider the following circuit containing a NPN BJT:



Suppose that $\beta = 100$, $V_1 = 6$ V, and $V_2 = 15$ V. Determine in which region the transistor is working and calculate the tensions V_{BE}, V_{CE}, V_{BC}, and the currents through the transistor.

Solution

Let's start by finding V_E. We know that $V_E = V_B - V_{BE}$, where V_{BE} is the potential drop across a diode. We can treat the diode as a perfect switch using the simplified DC model, which shifts the ideal diode behavior from an abrupt turn-on at 0 V to a threshold V_γ.

$$\begin{cases} I = 0 & if \quad V \leq V_\gamma \\ V = V_\gamma & if \quad I \geq 0 \end{cases}$$

Assuming I > 0, $V_{BE} = V_\gamma$. A good approximation is $V_\gamma \approx \frac{E_g}{2q}$. For silicon, this means about 0.6 V.
Consequently, since $V_B \equiv V_1$, V_E = 6 V - 0.6 V = 5.4 V.

$$I_E = \frac{V_E}{R_2} = \frac{5.4\ V}{3.3\ k\Omega} = 1.64\ mA$$

$$\beta \gg 1 \Rightarrow I_E \approx I_C = 1.64\ mA$$

$$V_{R1} = R_1 \cdot I_C = 4.7\ k\Omega \cdot 1.64\ mA = 7.7\ V$$

$$V_C = 15\ V - V_{R1} = 7.3\ V \Rightarrow V_{CE} = 1.9\ V > 0\ V$$

We now know we are in the direct forward region:

$$V_{BC} = 6\ V - 7.3\ V = -1.3\ V$$

$$I_B = \frac{I_C}{\beta} = 16.4\ \mu A$$